# *THE FOUR-POINT PROBLEM.*

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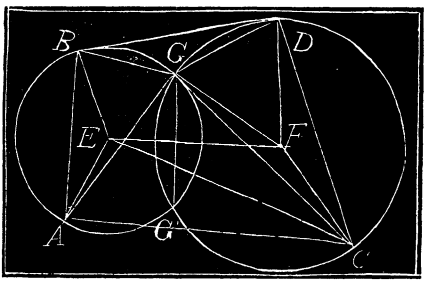
PROBLEM : – Four points in the same plane being given in position, to determine the position of any other point (or place of observation) in reference to these given points, having on the four points two angles which have no parts common except their vertices.




Let A, B, D, & C be the four given points: and let DGC & AGB be the observed angles. And then since the $\angle AEB = 2\angle AGB$, and $AE = BE$ $\therefore$ in $\triangle AEB$ we have BA and all the $\angle$s to find AE. And then in $\triangle$ EAC are known AE and AC and $\angle EAC$ to find EC and the $\angle$ ECA. But since $\angle DFC = 2\angle DGC$, and $DF = FC$ hence in $\triangle$ DFC we have DC and all the $\angle$s to get FC. But the $\angle FCE = \angle ACD - > ACE - > FCD$ $\therefore$ in $\triangle FCE$ are given FC and CE and the included angle to find FE and $>EFC$, then in $\triangle$ EGF all the sides are known, hence $>GFE$ can be found. But $>GFC = >EFC + >GFE$ $\therefore$ in $\triangle$ GFC we have GF and FC and all the angles to find GC. Then in $\triangle$ GCD are given GC, CD and $>CGD$ to get GD. And now GA and GB follow in a very obvious manner.

This problem may obviously be solved with fewer equations by using quadrilaterals, instead of triangles, in the solution.

Unless the two circles of position are tangent to each other there will be two points (viz, G and G′) which will equally well satisfy the conditions of the problem: hence in this case we judge from an approximate knowledge of our position which of the points G or G′ we were at when we observed.

The manner of sweeping the circles of position, with Protractor and Dividers is easy and expeditious: since $>AEB = 2>AGB$ then $>ABE$

$$= \frac{180^\circ - >AEB}{2} = \frac{180^\circ - 2>AGB}{2} = 90^\circ - AGB,$$

that is $>ABE =$ the complement of the observed $>AGB$. The rule for plotting the circles of position is now obviously, to lay off from AB at the points A, and B the compliment of the observed angle *A*GB, and the point of intersection of the produced sides of these angles will be

the center of a circle of position. And now from this point as center with radius AE (or BE) sweep the circle of position. And in like manner lay down the other circle of position through C and D.

This problem will often be found servicable to the Hydrographer and Explorer when from either accident or necessity only two angles are measured on four objects.

## *SOLUTIONS OF PROBLEMS IN NO. 6.*

Solutions of problems in No. 6 have been received as follows: From Geo. L. Dake, 25 & 26; R. M. DeFrance, 25 & 26; Prof. A. B. Evans, 25, 26, 27 & 29; Henry Gunder, 25, 26, 27 & 29; Wm. Hoover, 26; Prof. A. Hall, 29; H. Heaton, 29; D. J. McAdam, 25 & 26; Esther Matthews, 26; Artemas Martin, 27 & 29; A. W. Phillips, 25, 26, 27 & 29; L. Regan, 25, 26 & 29; R. L. Selden, 25; Werner Stille, 25, 26, 27 & 29; E. B. Seitz, 25, 26, 27 & 29; Prof. J. Scheffer, 26 & 29; Walter Siverly, 27.

25. "Required the sides of an obtuse angled triangle the area of which is 14.048 acres, the obtuse angle 111°15′, and one of the acute angles 11°44′10″."

SOLUTION BY HENRY GUNDER, GREENVILLE, OHIO.

Putting $A = 111°15'$, $B = 11°44'10''$, $C = 57°50''$, and $x$, $y$, $z$ or the sides opposite $A$, $B$ and $C$, and $a = 14.048$ acres $= 2247.68$ sq. rods.

Since the product of two sides and the sine of the included angle equals twice the area we have,

$$(1)\ xy = \frac{2a}{\sin C},\quad (2)\ xz = \frac{2a}{\sin B},\quad (3)\ yz = \frac{2a}{\sin A}.\quad \text{Then}$$

$$\sqrt{\frac{(1)\times(2)}{(3)}} = (4)\ x = \sqrt{\frac{2a\sin A}{\sin B\sin C}}.\quad \text{Similarly } y = \sqrt{\frac{2a\sin B}{\sin A\sin C}}.$$

$$\text{“}\quad z = \sqrt{\frac{2a\sin C}{\sin A\sin B}}.$$

By applying logarithms, $x = 156.705$ rods, $y = 34.1997$ rods, $z = 141.034$ rods.